114

Tropical Agricultural Research Vol. 2 1990

Mathematical Modelling of Tea Grading

A. Thevathasan and D.S.A. Samaraweera

Tea Research Institute
Talawakele

ABSTRACT: The size variation of stored tea was studied in relation to the particle size, composition of ungraded input tea dhools, and the mesh size of the sifter.

Sieve analysis technique was utilized in formulating a mathematical model to express the relationship between the input and throughput streams of a sifter. Using the results, a common mathematical formula was derived for mesh sizes No.8 (2032 microns) and No.18 (1184 microns) in relation to the particle sizes of the defined separation coefficient. The equation is $\lambda = \lambda_{max} (1 - P/P_o)$*, where:* λ_{max} *is a constant 0.0765, and* P_o *is the mesh size of an experimental sifter defined in terms of the equivalent aperture size of a standard sieve which varies with meshes.*

The concept of "separation coefficient" can be used to study the performance of the commercially used tea sifter for grading made tea offering an insight into the sieving operation.

INTRODUCTION

Tea grading is an important process in tea manufacture and is essentially a classification of the final product in terms of particle size. Particle size of tea dhools or tea particles is polydispersed (De Silva, 1972) and their variation in size is very large. For market acceptance, made tea particles should be seperated into uniform sizes. This is done mainly by sieve separation assisted by particle size reduction and cleaning.

Work is now underway in the International Standards Organization Sub Committee on Tea (ISO/TC 34/SC 8) to develop a system to classify the grades of black tea on the basis of retention of particles by sieves. This system merely defines the grades of tea but does not offer any insight into the sifting operation.

In this paper, a mathematical model has been formulated in relation to the input of tea dhools, the throughput and the mesh size of an experimental sifter to ascertain the reason for nonconformity of the sifted teas in relation to the particle size.

MATERIALS AND METHOD

The L.T.P (Lowrie Tea Processor) type of tea dhools was used for this experiment, due to its uniformity among the day to day samples. The feed rates to the experimental sifter were also kept constant at 1715 g/min. About 6 kg of dhools from the L.T.P. were collected on six different days to form six replicates.

The experimental sifter fitted with mesh No.8 (2032 microns) or with mesh No.18 (1184 microns). Six equal compartments were made on the collector box facilitating the collection of throughput in six equal areas. Initialy two, 100g representative samples from each of the six replicates were taken using the sample divider. Sieve analysis was carried out on these samples (Proft and Headly, 1976, Irani, 1961) to determine the particle size composition.

A 6 kg sample from each replicate was then sifted in succession in the experimental sifter. Throughput was collected and representative samples were taken from the six compartments and from the over size particles. All these samples which form a set were subjected to sieve analysis.

RESULTS AND DISCUSSION

The particle size analysis of the input and the throughput as collected in each compartment and the over flow-stream from a replicate are presented in Table 1. The above data were used to estimate the particle composition of the input and throughput in each of the segments of the sifter and similarly for all the other replicates. During the sifting in the experimental sifter the topmost corners of the first segment were not fully covered by the particles to maintain the same effective sieving area for all segments, and hence the reading from the first segment was discarded.

Table 1. Sieve analysis data of throughput (mesh No. 8) – L.T.P. dhools of a replicate

	Weight (g) of total input, throughput in each compartments and the oversize particles							
Particle size (microns)	Input	1	2	3	4	5	6	
1400	190.29	27.97	40.03	49.80	17.14	12.21	9.68	33.46
1200	461.57	126.16	167.19	126.12	16.66	8.50	5.18	14.21
1000	456.47	172.13	179.73	95.95	3.75	1.77	1.13	2.11
850	539.16	224.38	210.86	99.88	1.72	0.64	0.56	1.12
710	779.85	342.18	323.07	112.38	1.17	0.31	0.35	0.39
600	553.88	256.35	235.33	60.77	0.72	0.15	0.16	0.20
500	758.33	365.28	317.81	74.50	0.47	0.07	0.10	0.10
420	318.85	158.73	136.87	22.82	0.23	0.03	0.07	0.10
355	431.55	236.92	166.99	27.40	0.14	0.01	0.04	0.05
300	296.76	162.88	118.87	14.76	0.14	0.02	0.04	0.05

The input, throughput and particle size bands of a replicate are presented in Table 2. Linear regression analysis was carried out on these results considering the input and the throughput of the five segments as five different independent (X) and dependent (Y) variables respectively for the different particle sizes of 300, 355, 420, 500, 600, 710, 850, 1000, 1200 and 1400 microns. It is customary to quote particle sizes when sieving in terms of "sieve diameter" (snowsill, 1985) through which they were extracted. Sets of six linear lines representing each replicate for different particle sizes were obtained. Statistical analysis (Davies and Goldsmith, 1972) was carried out on all the data points from a set of six lines for a given particular size range and was found that the points lie on a common line. When the intercepts and the standard errors of intercepts were considered, the above appeared to almost pass through the origin as in ideal condition. The equation of the lines passing through the origin are recalculated and given in Table 3.

THEORY OF SEPARATION COEFFICIENT

Separation coefficient is defined as the ratio of the weight of the particles of a given particle size group separated through a single aperture of a sieve to that of the total weight of the particles of that particle size group apparently subjected to sieving in the same single aperture and can be derived as follows:

Let "n" and "N" be the number of aperture length wise and breadth-wise respectively in a sieve.
Let 'λ' be the separation coefficient for any one particular particle size group P.

Let 'M_{ip}' be the mass of the particle of that size group fed into the sieve and "M_{op}" be the total throughput of that sieve.
Therefore, M_{ip}/N is the total apparent weight of the particles subjected to sieving per length wise row.

Table 2. Sieve analysis data - calculated input and throughput (mesh No. 8) as collected in last five compartments of a replicate.

	Weight (g) of the particles									
Particle size (microns)	Compartments from feeding end									
	2nd		3rd		4th		5th		6th	
	Input	Through put	Input	Through put	Input	Through put	Input	Through put	Input	Through put
1400	162.32	40.03	122.29	49.80	72.49	17.14	55.35	12.21	43.14	9.68
1200	335.41	167.19	168.22	126.12	42.10	14.66	27.44	8.05	19.39	5.18
1000	284.44	179.73	104.71	95.95	8.76	3.75	5.01	1.77	3.24	1.13
850	314.78	210.86	103.92	99.88	4.04	1.72	2.32	0.64	1.68	0.56
710	437.67	323.07	114.60	112.38	2.22	1.17	1.05	0.31	0.74	0.35
600	297.53	235.53	62.00	60.77	1.23	0.72	0.51	0.15	0.36	0.16
500	393.05	317.81	75.24	74.50	0.74	0.47	0.27	0.07	0.20	0.10
420	160.12	136.87	23.23	22.82	0.43	0.23	0.20	0.03	0.17	0.07
335	194.63	166.99	27.64	27.40	0.24	0.14	0.10	0.01	0.09	0.04
300	133.88	118.87	15.01	14.76	0.25	0.14	0.11	0.02	0.09	0.04

Table 3. Equation of fitted lines through origin – Input Vs Throughput (mesh No. 8) at different particles sizes.

Particle size (microns)	Equations (X – Input and Y – Throughput in g)
300	Y = 0.8948 X
355	Y = 0.8744 X
420	Y = 0.8672 X
500	Y = 0.8440 X
600	Y = 0.8335 X
710	Y = 0.7876 X
850	Y = 0.7512 X
1000	Y = 0.7153 X
1200	Y = 0.5797 X
1400	Y = 0.3111 X

By definition the amount passing through the first aperture is given by:

$$(M_{1p}/N) * \lambda_p = O_1$$

Hence the amount subjected to sieving in the second aperture in the length wise direction is given by:

$$(M_{1p}/N) * (1 - \lambda_p)$$

Therefore the amount passing through second aperture is:

$$(M_{1p}/N) * (1 - \lambda_p) * \lambda_p = O_2$$

Similarly the amount subjected to sieving in the nth aperture is:

$$(M_{1p}/N) * (1 - \lambda_p)^{n-1}$$

The amount passing through the nth aperture is:

$$(M_{1p}/N) * (1 - \lambda_p)^{n-1} * \lambda_p = O_n$$

Therefore the amount of oversize particles at the end of the sifter is:

$$(M_{1p}/N) * (1 - \lambda_p)^{n-1} * (1 - \lambda_p)$$

$$= (M_{1p}/N) * (1 - \lambda_p)^n$$

But M_{op}/N = Initial input less oversize particles at the end.

i.e. $M_{op}/N = M_{1p}/N - (M_{1p}/N) * (1 - \lambda_p)^n$

Therefore,

$$M_{op} = M_{1p} (1 - (1 - \lambda_p)^n)$$

The present study showed a linear relationship between the input and the throughput streams for different particle size ranges and that this relationship is a set of straight lines passing through the origin.

Hence the slope b_p of the line for the paticular size particle group is given by the equation:

$$b_p = (1 - (1 - \lambda_p)^n)$$

From the above equation. knowing 'n' and the slope 'b_p', the separation coefficient for different particle sizes can be calculated. The relationship between separation coefficient (λ) and the particle size (P) was also found to be linear.

The equation thus obtained from the sieve analysis data for mesh No. 8 is given by:

$\lambda = -4\ 4225 \times 10^{-5}\ P + 0.0765$, where

λ - separation coefficient of particle size groups with respect to mesh size No. 8 and

P - aperture size of the standard sieve through which the particles were extracted.

A similar linear relationship, $\lambda = 8.3378 \times 10^{-5}P + 0.0733$, was obtained with mesh No. 18.

The upper limit of the separation coefficient λ_{max} and the limiting value of the particle P_o when P tends to zero can be estimated from the above two lines and is given below:

Mesh No./size	λ_{max}	P_o
8(2032 microns)	0.0765	1730
18(1184 microns)	0.0730	875

Hence the two lines can be re-cast in the form:

$\lambda = \lambda_{max}\ (1 - P/Po)$. However it was noticed from the above two lines that the values for λ_{max} were close to each other. An attempt was made to pool the data for both mesh Nos. 8 and 18 to find a linear relationship applicable to both. Statistical analysis (Davies and Goldsmith,

1972) showed that the single line best represented by the pooled data to be

$$\lambda = -0.0765\ (P/Po) + 0.0759$$

Considering the standard error of intercept (0.0409) the intercept value of 0.0759 can be adjusted as 0.0765 thus making the gradient and intercept to have the same numerical value. Hence the new line can be represented by:

$\lambda = \lambda_{max}\ (1 - P/Po)$ where $\lambda_{max} = 0.0765$ and Po is the "mesh size in terms of equivalent aperture of standard sieve" which is a function of the sifter mesh size.

The value λ_{max} can thus considered as constant and in dependent of the sizes of the mesh used in the experimental sifter. But this may vary for different sifters with different designs.

REFERENCES

Davies, O.L., and Goldsmith, P.L., (1972). Statistical methods in Research and Production with special reference to the Chemical Industry, I.C.I. Ltd., Oliver and Boyd.

De Silva, W.C.A., (1972). Unfolding particle size distribution of tea grade. 1 - High grown Broker Orange Pekoe and Broker Orange Pekoe Fannings, Tea Q. 43 (1 & 2) : 21 -35.

Irani, R.R., (1961). Particle Size Distribution Data. Cereal . Sci. Today 6:35.

Proft, G., and Headly, V., (1976). Methods of determining and expressing particle size. Feed Manufacturing Technol. : 512

Snowsill, W.L., (1985). Particle sizing Instrument Technology Vol. 1, "Mechanical Measurement". Butterworth, UK.: 146 - 161.